B.Sc (Sem-II) Maths Unit-1st Paper-IV, 14 रवि
Complex Analysis
Dr. Kaushal Kishor
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~~Theorem 4.~~

MORERA'S THEOREM. (1)

If $f(z)$ is a continuous function in a domain D and if for ~~every~~ any closed contour C in the domain D,

$$\int_C f(z)\,dz = 0,$$

then $f(z)$ is analytic within D.

[It is a ~~sort~~ of converse of Cauchy's theorem]

Proof :→ Let z_0 be a fixed point and z be a variable point inside the domain D. The value of the integral $\int_{z_0}^{z} f(t)\,dt$ is independent of the curve joining z_0 to z and depends on z only.

Let $F(z) = \int_{z_0}^{z} f(t)\,dt$

Let $(z+h)$ be a pt in the neighbourhood of z.

$$\therefore F(z+h) - F(z) = \int_{z_0}^{z+h} f(t)\,dt - \int_{z_0}^{z} f(t)\,dt$$

$$\text{Or, } F(z+h) - F(z) = \int_{z_0}^{z+h} f(t)\,dt + \int_{z}^{z_0} f(t)\,dt$$

$$= \int_{z}^{z_0} f(t)\,dt + \int_{z_0}^{z+h} f(t)\,dt$$

$$= \int_{z}^{z+h} f(t)\,dt$$

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(7)

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Or, $$\frac{F(z+h)-F(z)}{h} = \frac{1}{h}\int_{z}^{z+h} f(t)\,dt$$

Or, $$\frac{F(z+h)-F(z)}{h} - f(z) = \frac{1}{h}\int_{z}^{z+h} f(t)\,dt - f(z)$$

$$= \frac{1}{h}\int_{z}^{z+h} f(t)\,dt - \frac{f(z)}{h}\times h$$

$$= \frac{1}{h}\left[\int_{z}^{z+h} f(t)\,dt - f(z)\int_{z}^{z+h} dt\right]$$

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$$= \frac{1}{h}\left[\int_{z}^{z+h} \{f(t) - f(z)\}\,dt\right]$$

$$\therefore \left|\frac{F(z+h)-F(z)}{h} - f(z)\right| = \left|\frac{1}{h}\int_{z}^{z+h}\{f(t)\,\cancel{dt} - f(z)\}\,dt\right|$$

$$\leq \frac{1}{|h|}\int_{z}^{z+h} |f(t) - f(z)|\,|dt|$$

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Or, $\left| \dfrac{F(z+h)-F(z)}{h} - f(z) \right| < \dfrac{\epsilon}{|h|}|h|$

$\left[\because |f(t)-f(z)| < \epsilon \right.$ for $|t-z| < \delta$ as $f(z)$ is continuous$\left.\right]$

$\therefore \left| \dfrac{F(z+h)-F(z)}{h} - f(z) \right| < \epsilon$ but $\epsilon \to 0$

$\therefore \dfrac{F(z+h)-F(z)}{h} - f(z) = 0$

$\lim_{h\to 0} \left\{ \dfrac{F(z+h)-F(z)}{h} - f(z) \right\} = 0$

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Or, $\lim_{h\to 0} \dfrac{F(z+h)-F(z)}{h} = f(z)$

Or $F'(z) = f(z)$

Thus derivative of $F(z)$ exists and so $F(z)$ is analytic in D. But we know derivative of analytic function is analytic. Thus $F'(z)$ i.e. $f(z)$ is analytic in D.

Proved

Thus derivative of $F(z)$ exists ~~and~~ so $F(z)$ is analytic in D.

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We know that derivative of analytic function is also analytic.

Hence $F'(z)$ is also analytic in D

But $F'(z) = f(z)$

$\therefore f(z)$ is also analytic in D.